TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 771

TRANSIENT EFFECTS OF THE WING WAKE ON THE HORIZONTAL TAIL

By Robert T. Jones and Leo F. Fehlner
Langley Memorial Aeronautical Laboratory

Washington
August 1940

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 771

TRANSIENT EFFECTS OF THE WING WAKE ON THE HORIZONTAL TAIL

By Robert T. Jones and Leo F. Fehlner

SUMMARY

An investigation was made of the effect of the wing wake on the lift of the horizontal tail surfaces. In the development of expressions for this effect, the growth of wing circulation and wing wake, the time interval represented by the tail length, and the development of lift by the tail were considered. The theory has been applied to a specific case to show the magnitude of the effect to be expected.

It is shown that, for motions below a certain frequency, the development of lift by the tail may be represented by a simple lag function. The lag is, however. somewhat greater than that indicated by the tail length.

INTRODUCTION

During unsteady motions of the airplane, the wing leaves in its wake a sheet of vortices of varying strength. The velocity induced by these vortices may have a pronounced effect on the direction of the air flow near the tail, particularly during motions involving rapid changes of lift such as oscillations of short period or passage through gusts.

An approximation to the effect of the wing wake has been used by Cowley and Glauert (reference 1). They assumed that the downwash associated with a change in lift is equal to the corresponding steady value but that the effect at the tail is delayed by the time required for the airplane to travel a distance equal to the tail length. It is known, however, that during increases of circulation the wing develops counterrotating vortices which must, for a time, at least, induce a strong upwash, increasing the lift of the tail. In addition to the time lag considered by Cowley and Glauert, both the variation of vertical velocity and the delay in the development of lift by the tail are considered in the present paper.

The flow around the wing, hence the wake produced by the wing, is assumed to be uninfluenced by the presence of the tail surface. The interference is thus confined to the effect of the wing on the tail and, since the wake formed by an isolated wing is known, the interference can be directly calculated for any relative position of the two surfaces.

Although the theory is thus applicable to a variety of arrangements, computations to cover all conditions were not considered to be worth while. In particular, the exact vertical location of the tail surface (within the usual range) was not expected to be critical. The tail surface was therefore considered to be located directly in the wake where the effect is a maximum. The effect of tail length was investigated and it was found that the results obtained from computations covering a typical case could be extrapolated to take account of this factor in a satisfactory manner.

LIFT FUNCTIONS

As in other problems involving unsteady flow, it is convenient to assume at first a sudden unit jump in the angle of attack of the wing and to develop more general solutions by operational methods (reference 2). The lift of the tail surface under these conditions is due solely to interference from the wing and is to be added to the lift independently developed.

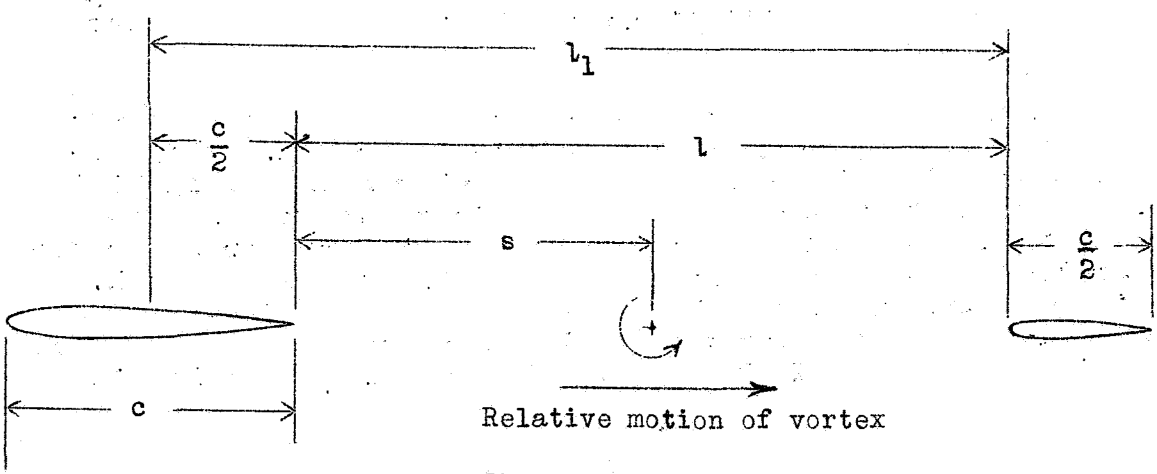

Figure 1.

Figure 1 shows the notation used in the development. Variable quantities are expressed in terms of the distance along the flight path, $s = \frac{U_o t}{c/2}$, where t represents time. The analysis is kept nondimensional by expressing all velocities in terms of the flight velocity, U_o, and all lengths in terms of the half chord, $c/2$.

The rate of development of vorticity by the wing following a sudden unit increase in angle of attack α ($\alpha = 1(s)$) is given in reference 3 and shows directly the distribution of vortices in the wake. The total circulation at any instant after such a change is denoted by $\Gamma_\alpha(s)$. The vertical velocity of air in the vicinity of the tail induced by a unit wake vortex, $w_\Gamma(s)$, (fig. 2) may be calculated from the Biot-Savart rule. The resultant variation of vertical velocity following a unit change of angle of attack of the wing follows from the combination of these two functions, i.e.,

$$w_w(s) = w_\Gamma(s)\, \Gamma_\alpha(0) + \int_o^s w_\Gamma(s - s_o)\, \Gamma_\alpha'(s_o)\, ds_o \qquad (1)$$

The effect of the vertical velocity, w_w, on the tail surface may be treated as the effect of a varying gust. The lift on an airfoil penetrating this gust is given by

$$C_{L_{t_w}}(s) = C_{L_g}(s)\, w_w(0) + \int_o^s C_{L_g}(s-s_o)\, w_w'(s_o)\, ds_o \qquad (2)$$

where $C_{L_g}(s)$ is the gust lift function for the tail surface (reference 3).

The integrations (1) and (2) may be carried out in a single step by using the operational equivalents of the functions involved. Thus

$$C_{L_{t_w}}(s) = \overline{C}_{L_{t_w}}(D)\, 1(s) = \overline{C}_{L_g}(D)\, \overline{\Gamma}_\alpha(D)\, \overline{w}_\Gamma(D)\, 1(s) \qquad (3)$$

The function $C_{L_{t_w}}(s)$ is the lift on the tail surface due solely to a unit change in the angle of attack of the wing and is to be added to the lift developed independently by the tail surface.

The function $\Gamma(s)$ denotes the circulation around the wing (or the equal and opposite circulation measured around the wake) in the plane of symmetry. The spanwise distribution of vorticity is assumed to remain elliptical. A unit increment of $\Gamma(s)$ thus involves a single wing and wake vortex of unit strength at the center, the strength falling off toward the tips in accordance with the elliptic loading. The two vortices are connected by a sheet containing only the spanwise component of discontinuity. This arrangement can be derived by the superposition of vortices of the type shown in figure 3.

The centroid of wing circulation is assumed to remain stationary at the center of the wing chord. Although the wing circulation originates at the trailing edge, little error is incurred through this assumption because the travel of the centroid to the center of the chord is very rapid (reference 3). If the wing circulation is replaced by a single vortex A (fig. 3), the vertical velocity at the tail due to vortices A and B is given by

$$w_{\Gamma_{AB}}(s) = \frac{1}{2\pi}\left(\frac{1}{l - s} - \frac{1}{l_1}\right) \tag{4}$$

The operational equivalent of this function is

$$\bar{w}_{\Gamma_{AB}}(D) = \frac{1}{2\pi}\left[D\, e^{-lD}\, \mathrm{Ei}(lD) - \frac{1}{l_1}\right] \tag{5}$$

where the symbol Ei represents the exponential integral function, i.e.,

$$\mathrm{Ei}(x) = \int_{\infty}^{-x} \frac{e^{-u}}{u}\, du$$

The downwash due to the spanwise component of discontinuity (vortices C and D, fig. 3) may be determined from figure 6 of reference 3, which shows the downwash at the edge of a sheet of discontinuity of varying length. The downwash in the region of the tail corresponding to any position or extent of the wake may be obtained by adding the effects of two sheets of different lengths, as indicated in figure 4.

APPLICATION OF THEORY

In order to illustrate the application of the theory and to show the order of magnitude of the results to be expected, the lift functions are determined for a specific example. The proportions considered are as follows. (See fig. 1.)

Aspect ratio of wing - - - - -	6
Aspect ratio of tail surface - -	3
l - - - - - - - - - - - - - - -	5.54
l_1 - - - - - - - - - - - - - -	6.54
Chord of tail surface - - - - -	1.0

The effect of the noncirculatory component of the flow about the wing is neglected, its influence at the tail being small and constant in value.

The approximate expressions used for the functions $\overline{C}_{L_g}(D)$ and $\overline{\Gamma}_\alpha(D)$ are:

$$\overline{C}_{L_g}(D) \cong 3.77 - \frac{2.56\ D}{D + 1.116} - \frac{1.044\ D}{D + 6.40} \tag{6}$$

$$\overline{\Gamma}_\alpha(D) \cong 4.71 - \frac{2.11\ D}{D + 0.290} - \frac{1.25\ D}{D + 0.690} - \frac{0.800\ D}{D + 0.276} \tag{7}$$

It should be noted that the function C_{L_g} is in terms of the half-wing chord which, in the example chosen, is twice the corresponding dimension of the tail surface itself. There is a slight change in the expression as given in reference 3 to make it more closely approximate the starting value.

If the proper values are substituted in equations (4) and (5), the function $w_{\Gamma_{AB}}$ becomes

$$w_{\Gamma_{AB}}(s) = \frac{1}{2\pi}\left(\frac{1}{5.54 - s} - \frac{1}{6.54}\right) \tag{8}$$

and

$$w_{\Gamma_{AB}}(D) = \frac{1}{2\pi}\left[D\ e^{-5.54D}\ Ei(5.54D) - \frac{1}{6.54}\right] \tag{9}$$

No concise expression for the spanwise component of the downwash, $w_{\Gamma_{CD}}$, can be given. The curve can be approximated, however, by a series of exponential terms of the form $Ae^{\lambda s}$. For the proportions considered

$$w_{\Gamma_{CD}}(s) \cong -0.083 + 0.047e^{-0.067S} + 0.145e^{-0.354S}$$

$$-0.156e^{-0.742S} + 0.047e^{-1.45S} \tag{10}$$

$$\overline{w}_{\Gamma_{CD}}(D) \cong -0.083 + \frac{0.047\ D}{D + 0.067} + \ldots \tag{11}$$

These expressions are to be added to equations (8) and (9), respectively, to give the function $\overline{w}_{\Gamma}(D)$ required in the evaluation of equation (3).

The calculation of $C_{L_{t_w}}$ according to equation (3) results in the lift of the tail surface, as a function of the distance traveled following a sudden unit jump in the angle of attack of the wing, i.e., the "indicial lift"; it is shown in figure 5.

For a unit change in angle of attack of the airplane as a whole, the lift developed independently by the tail must be added. This lift increment is given by equation (29) of reference 3 for an airfoil of aspect ratio 3 but it must be expressed in terms of the wing chord. The function $C_{L_{t_\alpha}}(s)$ (fig. 6) shows the lift resulting from the unit change of angle of attack of the entire airplane.

Although the indicial lift curves (figs. 5 and 6) show infinite values, it is to be noted that the integration of the expressions by superposition for any probable disturbance results in finite lift at all points. Furthermore, when the effects of moderate rates of change in the angle of attack are integrated, the exact form of the indicial response curve is not critical.

This point is best illustrated by integrating the response to a continuous oscillatory variation of angle of attack. If $\alpha = e^{ins}$,

$$C_{L_{(\alpha=e^{ins})}}(s) = \overline{C}_L(D)\ e^{ins} = \overline{C}_L(in)\ e^{ins} = \left[A(n)+iB(n)\right]\ e^{ins}$$

The real part, A, is the component of the response in phase with the disturbance, and the imaginary part, B, is the component 90° out of phase with the disturbance.

The evaluation of the exponential integral with an imaginary argument is given in reference 4, page 80.

$$Ei(in) = Ci(n) + i\left[Si(n) + \frac{\pi}{2}\right]$$

where Ci and Si are, respectively, the cosine-integral and the sine-integral functions.

Figure 7 shows the A and the B components of the oscillatory lift function, $C_{L_{t_w}}(in)$, the lift on the tail surface induced by a continuous sinusoidal oscillation of the wing. For a continuous vertical oscillation of the airplane as a whole (changes of angle of attack without rotation), the function $C_{L_{t_\alpha}}(in)$ (fig. 8) shows the resulting lift.

If the oscillatory lift functions are approximated by a Fourier series, this series will be found to correspond to an approximation of the indicial lift function in the form of steps. Thus the function $C_{L_{t_w}}(in)$ is closely approximated, as shown by broken lines in figure 7, by the expression

$$C_{L_{t_w}}(in) \cong 0.30 - 2.20\ e^{-7.14in} \qquad (12)$$

for values of n less than 0.35. If the argument (in) is replaced by the operator D, the resulting function is the operational equivalent of a simple step function, which is an approximation of the corresponding indicial lift function. Thus

$$C_{L_{t_w}}(s) \cong \left[0.30 - 2.20\ e^{-7.14D}\right] 1(s) \cong 0.30 - 2.20\left[1(s - 7.14)\right] \qquad (13)$$

This approximation is shown by the broken line in figure 5.

Similarly, the approximation shown by the broken line in figure 8 is

$$C_{L_{t_\alpha}}(in) \cong 3.85 - 1.98\ e^{-7.48in} \tag{14}$$

and

$$C_{L_{t_\alpha}}(s) \cong \left[3.85-1.98\ e^{-7.48D}\right] 1(s) \cong 3.85-1.98\ \left[1(s-7.48)\right] \tag{15}$$

and is shown by the broken line in figure 6.

In the case of an airplane executing pitching motions during which the angle of attack of the wing does not change, the component of response out of phase with the disturbance is insignificant. The response may therefore be considered instantaneous. Although the general case involves motions that combine changes of angle of attack, α, and of angular displacement, θ, the lift increments resulting from each motion have been separately treated. In this form, the results are directly applicable to the differential equations of motion of the airplane.

CONCLUDING REMARKS

Although the indicial lift of the tail surface actually shows a pronounced variation, it is permissible to consider the effect of a simple lag if only moderate rates of motion are involved. The lag functions shown in figures 5 and 6 give the lift quite accurately during any motion that can be compounded of frequencies lower than one cycle in 18 half chords $(n < 0.35)$.

It will be noted that these expressions differ from those assumed by Cowley and Glauert in two ways. First, the value of the function from 0 to l is not zero but is a positive value, which accounts for the upwash that the tail initially encounters. Second, the distance after which the value of the function becomes negative is not equal to the tail length, l, but occurs at a distance somewhat greater than the tail length. This distance accounts for the lag in the growth of downwash and the lag in the development of lift by the tail and may be called the effective tail length.

Although the results thus far have been obtained for one specific wing and tail arrangement, the effective tail length and the magnitude of the effect to be expected may be determined for other cases.

According to the theory, the pattern of the wake formed by the wing remains unchanged as it passes downstream. In addition, the rapid changes of induced velocity at the tail occupy a fairly short distance immediately ahead of and back of the edge of the wake. For any usual tail length, therefore, the time history of the lift on the tail is not substantially altered in relation to the instant at which the wake strikes the tail. This point is illustrated by figure 9, where the induced vertical velocity, w_Γ, following a unit change in wing circulation $(\Gamma = 1(s))$ is shown for different positions of the wing ahead of the tail surface. The principal effect of a change in tail length is to shift the origin along the relatively flat portion of the indicial lift curves shown in figures 5 and 6. This change in tail length then corresponds to an equal change in effective tail length. The effective tail length for any case is thus determined.

If the wing wake passes either above or below the tail surface, the lift functions will not show infinite values as they do in the case considered. The peak value of the lift function is lowest when the wake passes below the tail surface. The final value of the lift is but little affected by the vertical displacement as long as this displacement is small relative to the span.

The theory may be extended to show the effects of vertical gusts. The lift of the tail surface due to interference from the wing during penetration of the gust may be calculated with the aid of the curves of figure 5, provided that the variation of the angle of attack of the wing is known. A close approximation to this variation of angle of attack during penetration of a varying gust is obtained by measuring the angle of attack with respect to the relative wind direction at a point one-fourth of the chord ahead of the trailing edge of the wing. This approximation is based on a well-known result of the thin-airfoil theory and is valid as long as the rate of change of gust velocity along the flight path is less than that represented by an oscillation of one cycle in 18 chord lengths. The lift independently developed by the action of the gust on the tail surface may be determined from reference 3. This lift is directly added to that developed by interference from the wing.

As noted earlier, only in cases of extremely rapid changes in wing lift ($n > 0.35$) is the exact form of the indicial lift curve important. Such changes may occur, however, in sharp gusts and it is believed that the extension of the present investigation to cover the effect of vertical position of the tail surface in these cases would be worth while.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., July 23, 1940.

REFERENCES

1. Cowley, W. L., and Glauert, H.: The Effect of the Lag of the Downwash on the Longitudinal Stability of an Aeroplane and on the Rotary Derivative M_q. R. & M. No. 718, British A.R.C., 1921.

2. Jones, Robert T.: Operational Treatment of the Nonuniform-Lift Theory in Airplane Dynamics. T.N. No. 667, NACA, 1938.

3. Jones, Robert T.: The Unsteady Lift of a Wing of Finite Aspect Ratio. T.R. No. 681, NACA 1940.

4. Jahnke, Eugen, and Emde, Fritz: Tables of Functions. 2d ed., B. G. Teubner (Leipzig and Berlin), 1933.

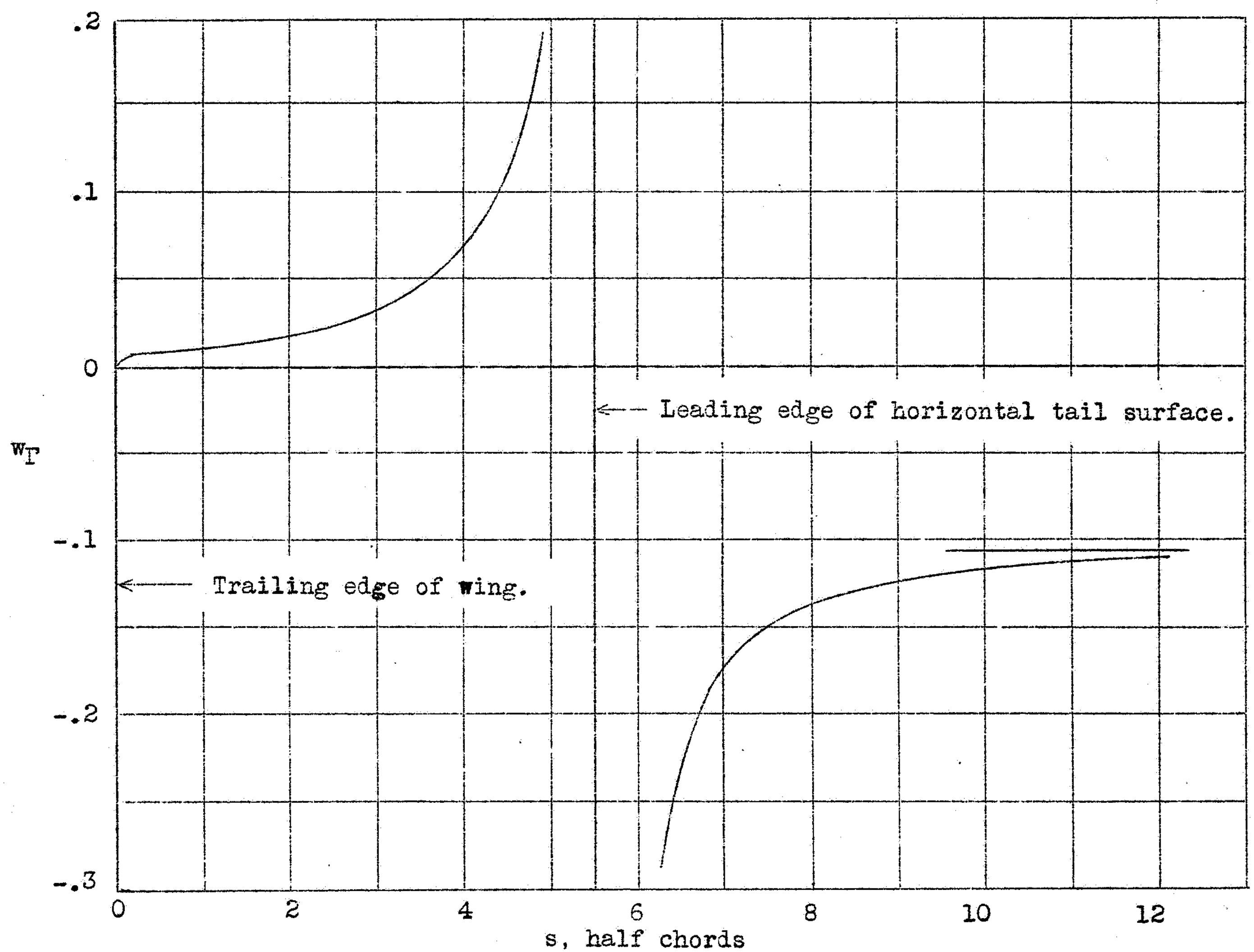

Figure 2.- Variation of vertical velocity at the horizontal tail following a unit increase in wing circulation, $w_\Gamma(s)$.

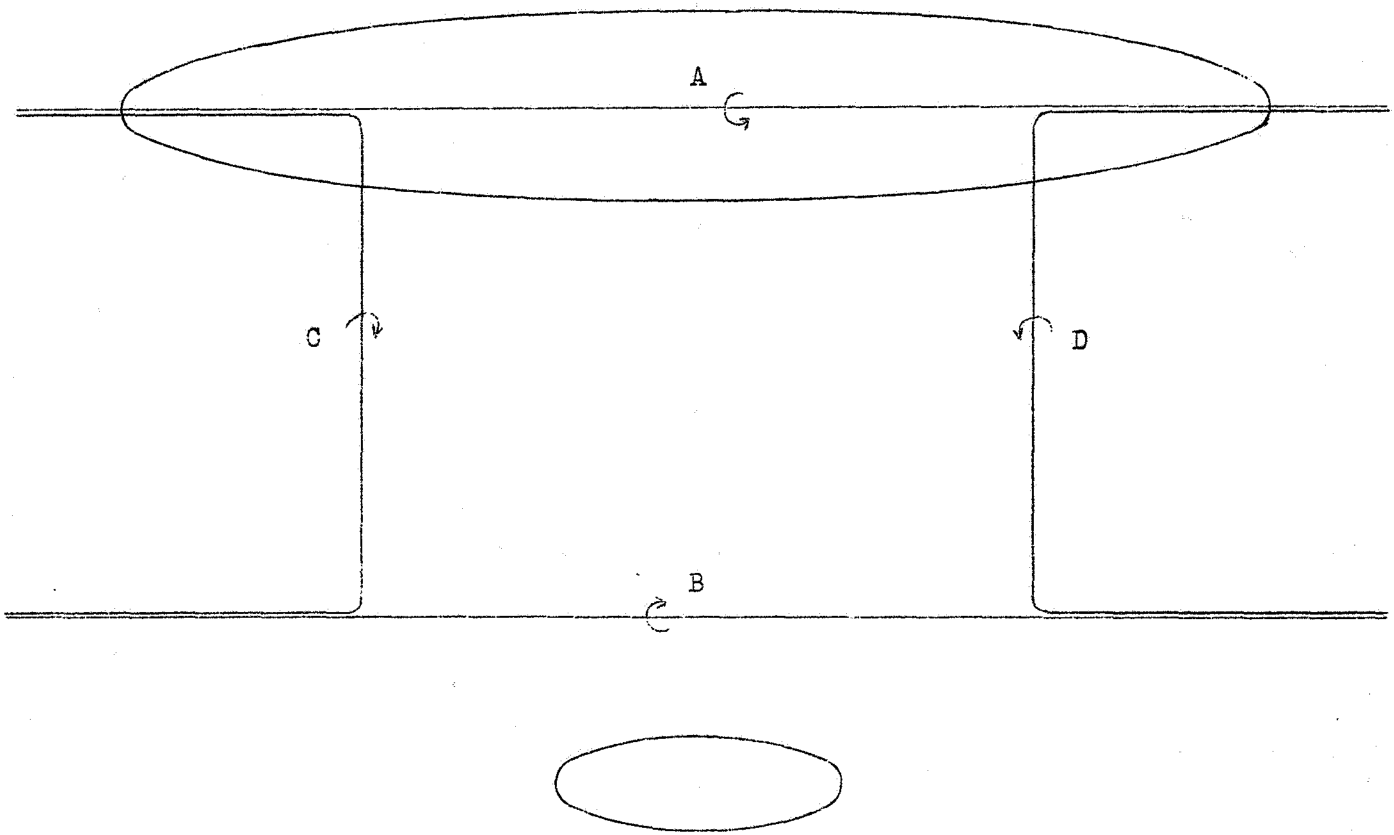

Figure 3.- Superposition of vortices to obtain finite loop.

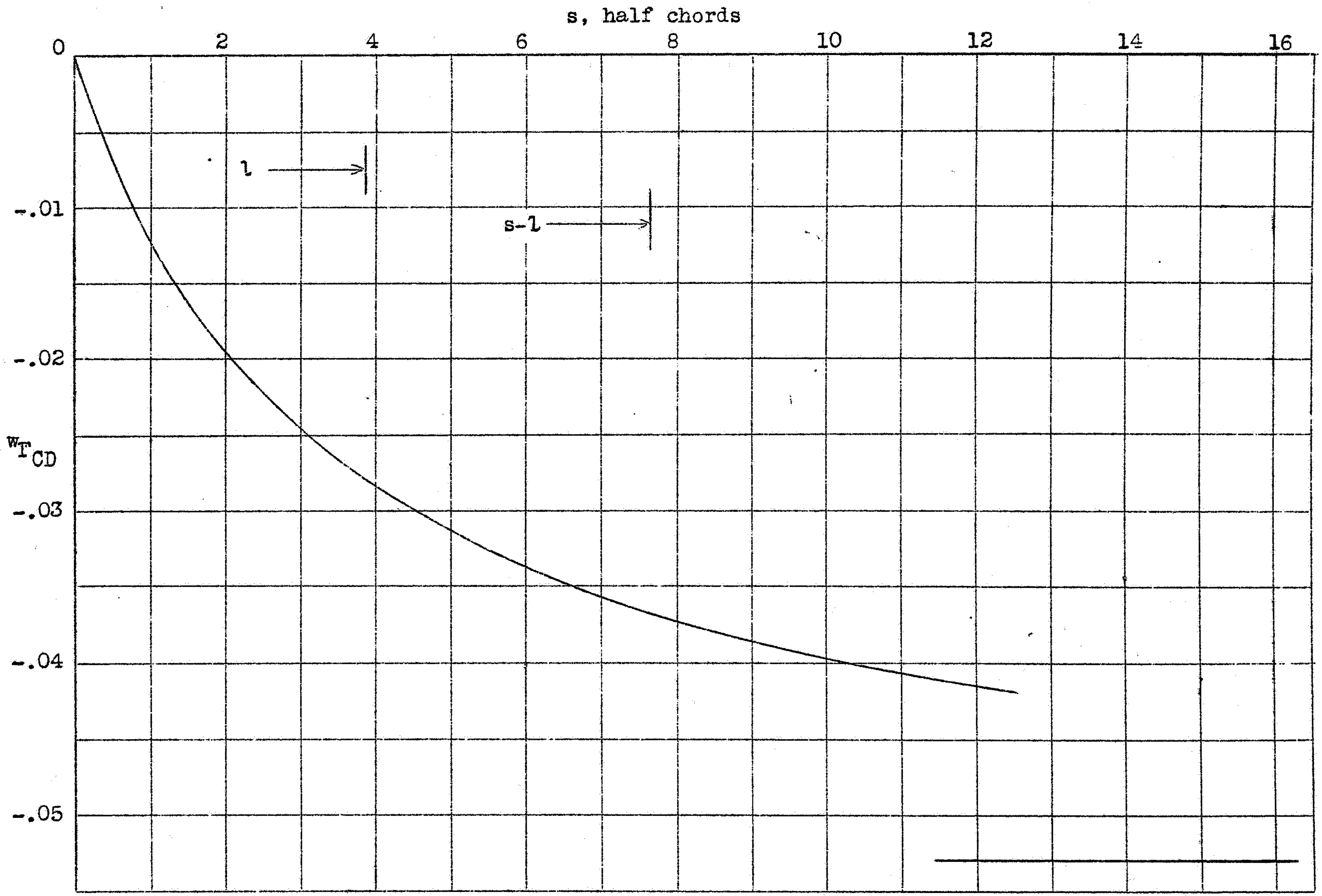

Figure 4.- Variation of downwash in the plane of symmetry with increasing length of the wake. Spanwise component of discontinuity.

$$w_{\Gamma_{CD}}(s) = w_{\Gamma_{CD}}(l) + w_{\Gamma_{CD}}(s-l)$$

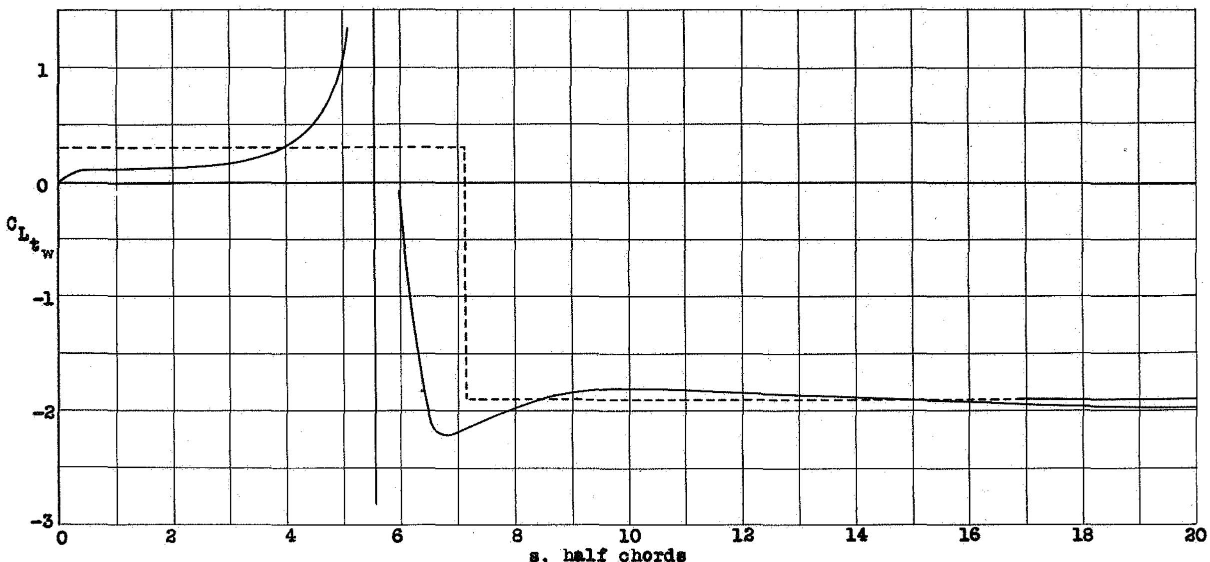

Figure 5.- Variation of lift on horizontal tail following an instantaneous unit change of angle of attack of the wing.

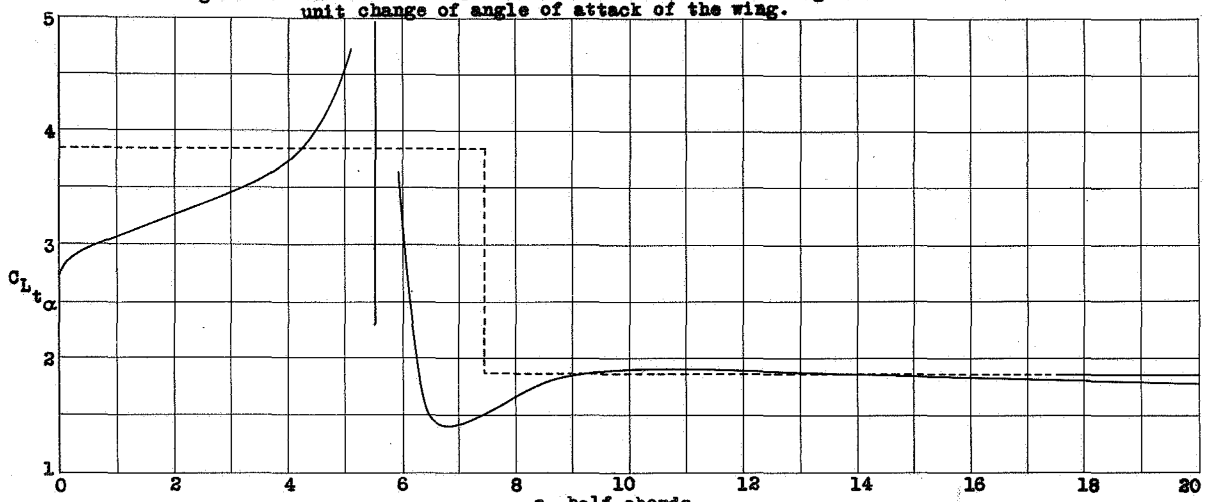

Figure 6.- Variation of lift on horizontal tail following an instantaneous unit change of angle of attack of the airplane.

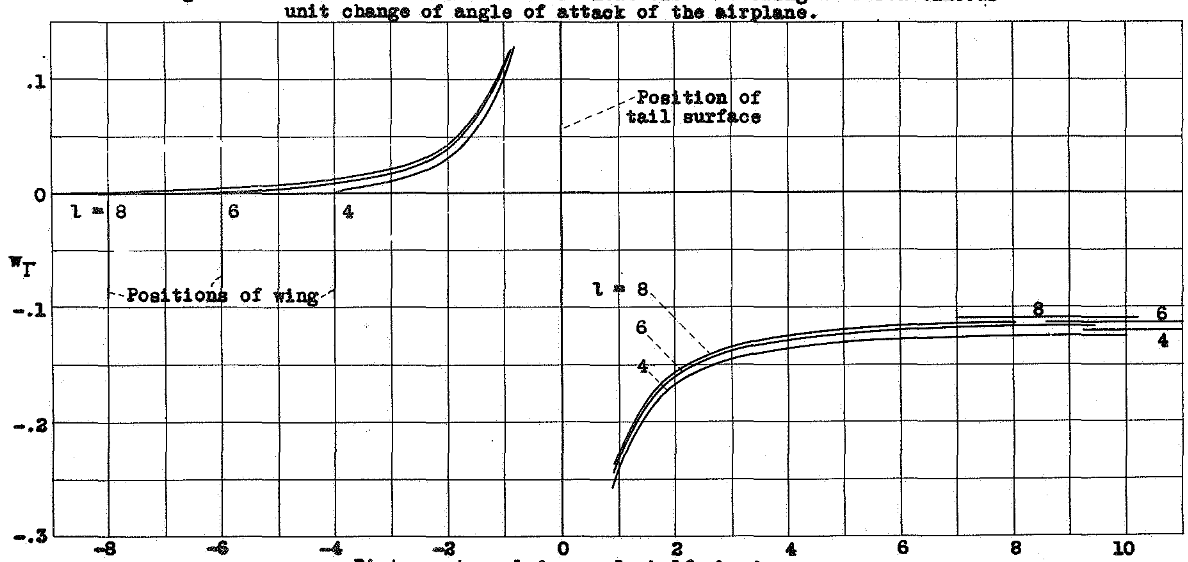

Figure 9.- Effect of tail length on the vertical velocity at the horizontal tail following a unit increase in wing circulation, $w_\Gamma(s)$.

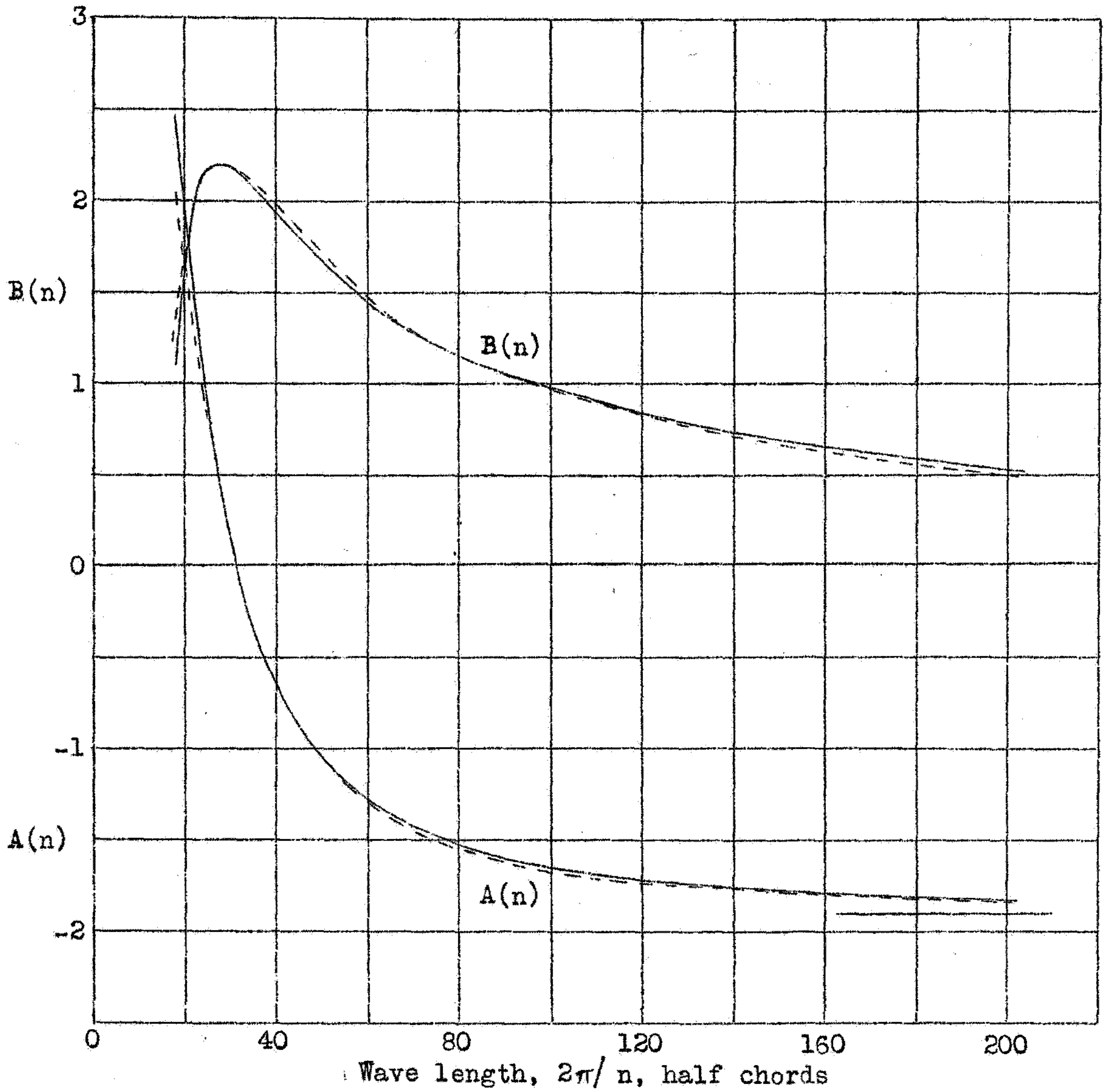

Figure 7.- Influence of wing oscillation on the lift of the horizontal tail. $C_{L_{t_w}}(in) = A(n) + iB(n)$

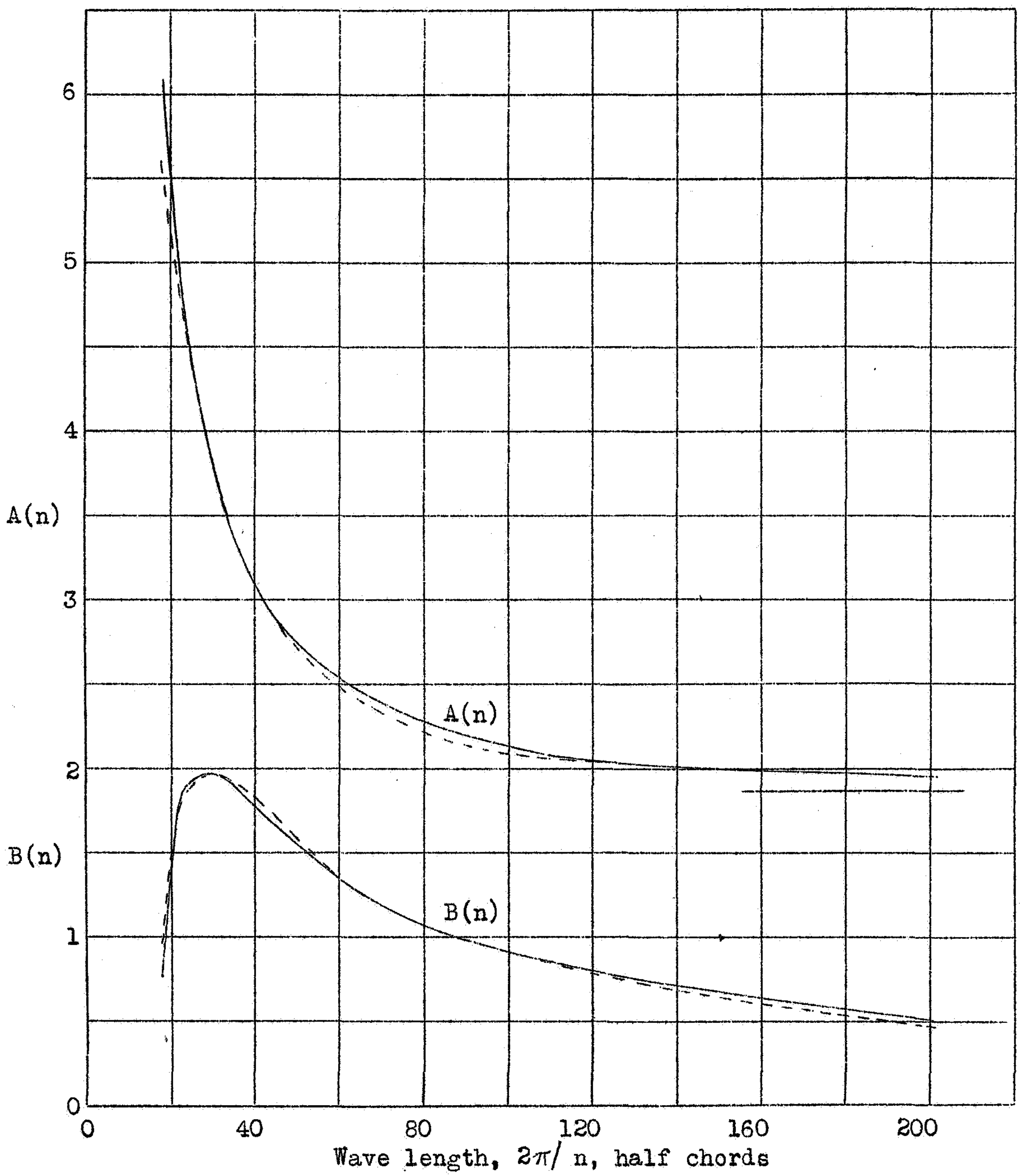

Figure 8.- Lift functions for horizontal tail of airplane in vertical oscillation without pitching. $C_{L_{t_\alpha}}(in) = A(n) + iB(n)$